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normalization and soft decision Viterbi  
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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



## THESIS

ERROR PROBABILITIES OF FFH/BFSK WITH  
NOISE NORMALIZATION AND SOFT DECISION  
VITERBI DECODING IN A FADING CHANNEL  
WITH PARTIAL-BAND JAMMING

by

Anthony Y. P. Chua

March 1994

Thesis Advisor:  
Thesis Co-Advisor:

Tri T. Ha  
R. Clark Robertson

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Error Probabilities of FFH/BFSK with Noise Normalization  
and Soft Decision Viterbi Decoding in a Fading Channel  
with Partial-Band Jamming

by

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Submitted in partial fulfillment  
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## ABSTRACT

An error probability analysis of a communications link employing convolutional coding with soft decision Viterbi decoding implemented on a fast frequency-hopped, binary frequency-shift keying (FFH/BFSK) spread spectrum system is performed. The signal is transmitted through a frequency non-selective, slowly fading channel with partial-band jamming. Noise normalization combining is employed at the receiver to alleviate the effects of partial-band jamming. The received signal amplitude of each hop is modeled as a Rician process, and each hop is assumed to fade independently.

It is found that with the implementation of soft decision Viterbi decoding that the performance of the receiver is improved dramatically when the coded bit energy to partial-band noise power spectral density ratio ( $E_b/N_j$ ) is greater than 10dB. At higher  $E_b/N_j$ , the asymptotic error improves dramatically and varies from  $10^{-6}$  to  $10^{-12}$  depending on the constraint length ( $\nu$ ), number of hops/bit ( $L$ ), and the strength of the direct signal ( $\alpha^2/2\sigma^2$ ). In addition, nearly worst case jamming occurs when the jammer uses a full band jamming strategy, even when  $L=1$  and there is a very strong direct signal ( $\alpha^2/2\sigma^2 = 100$ ). Due to non-coherent combining losses, when the hop per bit ratio is increased, there is some degradation at moderate  $E_b/N_j$ . Furthermore, when a stronger code is used (i.e., the constraint length is longer), performance improves, especially for high  $E_b/N_j$  where the asymptotic error is reduced. Finally, soft decision decoding improves performance over hard decision decoding from 4 to 8dB at moderate  $E_b/N_j$  depending on the code rate and with a much lower asymptotic error at high  $E_b/N_j$ .

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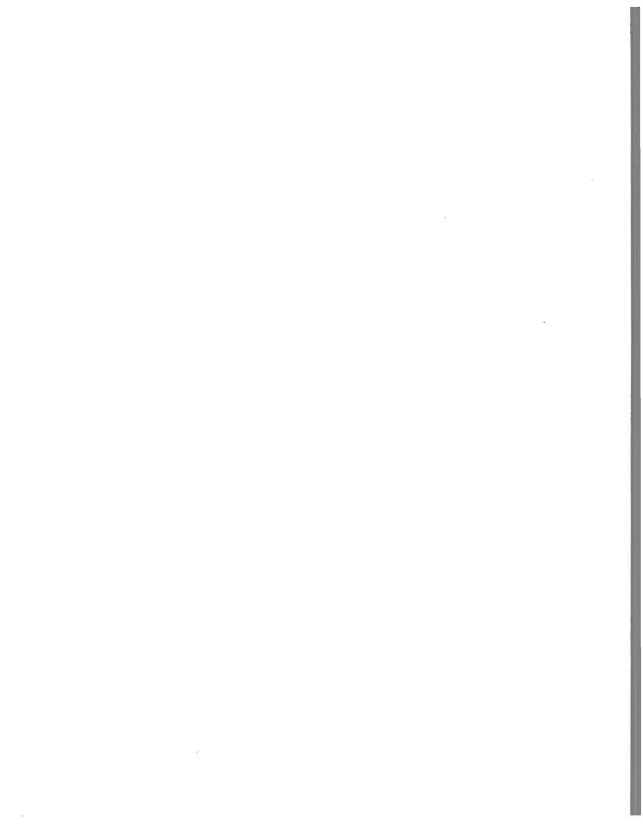
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## LIST OF SYMBOLS

$\alpha^2$	Average Power of the Non-faded (direct) component of the signal
$2\sigma^2$	Average Power of the Rayleigh-faded (diffuse) component of signal
$\sigma_k^2$	Average Power of the Noise at hop k
$\gamma$	Fraction of hops jammed
$\lambda$	$(2\sigma^2 + \sigma_k^2) / (2\sigma_k^2 \sigma^2)$
$\psi$	$\sqrt{\{(2z_{ik}) / \sigma_k\}}$
$\zeta$	$\alpha^2 / \sigma^2$
$\xi_k$	$2\sigma^2 / \sigma_k^2$
$\rho_k$	$\alpha^2 / \sigma_k^2$
$\beta_k$	$1 / (2(1 + \xi_k))$
$\omega_j$	Number of paths for event of weight j
$A_i^n$	$n(n-1)(n-2).....(n-i+1)$
$a_k$	RMS amplitude of signal
B	Bandwidth of the signal
$d_{free}$	Hamming free distance
$E_b$	Uncoded bit energy
$E_c$	Coded bit energy
$N_i$	Power Spectral Density of the Jamming Noise
$N_0$	Power Spectral Density of the Thermal Noise
$N_T$	Power Spectral Density of the Total Noise
$P_b$	Probability of Bit error

$P_j$	Probability of an error event of weight $j$
$P_j(i)$	Conditional probability of an error event given that $j$ hops are jammed
$P_T$	Total Noise Power
$R_b$	Information Bit Rate
$R_h$	Hop Rate
$X_{1k}, X_{2k}$	Received Signal at Branch 1 and 2
$Z_{1k}, Z_{2k}$	Normalized received signal at Branch 1 and 2



## I. INTRODUCTION

An important characteristic of military communications is the requirement to be robust against hostile interference or other unintentional interference. Fast frequency-hopping spread spectrum systems are widely used in military communications to mitigate the effects of hostile interference. Fast frequency-hopping implies that each bit is transmitted multiple times at different carrier frequencies determined by some pseudorandom sequence known only by the transmitter and intended receiver. As a result, the hostile interference must be spread over a much wider bandwidth which results in a lower interference noise power spectral density.

Fast frequency-hopped systems are susceptible to partial-band interference when the demodulator assigns equal weight to each of the multiple transmissions, or hops. As a result, various methods of signal normalization to overcome the adverse effects of partial-band jamming have been considered. Previous works have studied the performance of receivers utilizing noise normalization [Ref. 1], ratio-statistic normalization [Ref. 2], and self-normalization [Ref. 3].

For receivers utilizing noise normalization, convolutional coding with hard decision decoding [Ref. 1] has been studied. It was found that with hard decision decoding that performance is improved significantly as compared to the uncoded system. The implementation of soft decision decoding is expected to further improve performance [Ref. 4]. The analysis and actual implementation of a receiver utilizing soft decision



decoding requires much more computing power than a receiver with either no coding or one with hard decision decoding since the decision on the coded transmissions are not made until they are in the Viterbi decoder. With improved computing capability, soft decision convolutional coding is now practical and will be implemented in the new generation of frequency-hopping radios. The investigation of the performance of systems using convolutional coding and soft decision Viterbi decoding is an important problem as such systems will be installed in satellite and other military systems.

An error probability analysis of a communications link employing convolutional coding with soft decision Viterbi decoding of a fast frequency-hopped, binary frequency-shift keying (FFH/BFSK) spread spectrum system with non-coherent, noise-normalized detection and a channel with Rician fading and partial-band jamming is performed in this thesis. This is a continuation of the research conducted in [Ref. 1] with the implementation of convolutional coding with soft decision Viterbi decoding.

The data are first encoded by convolutional coding. After that, the transmitter transmits  $L$  hops for each encoded bit. The channel is modeled as a Rician fading channel. Both partial-band interference and thermal noise are also assumed to affect the channel. The partial-band interference is assumed to be unaffected by the fading channel.

The FFH/BFSK receiver with noise normalization and soft decision Viterbi decoding is shown in Fig. 1. At the receiver, perfect dehoppping is assumed and the signals are demodulated by a pair of quadrature receivers. Before going to the soft-decision Viterbi decoder, the signals are passed through a noise normalization combiner.

At the noise normalization combiner, the noise power, estimated using a noise-only channel estimator, is inverted and used to multiply each hop of an encoded bit.

Each hop is assumed to fade independently, and the channel for each hop is modeled as a frequency non-selective, slowly fading Rician process. Given these assumptions, the implication is that the channel's coherence bandwidth is smaller than the smallest frequency hop spacing but larger than the dehopped signal bandwidth, and the channel coherence time is larger than the hop duration. The amplitude of the dehopped signal is modeled as a Rician random variable and consists of a non-faded (direct) component and a Rayleigh-faded (diffuse) component [Ref. 4].

There are two types of interference considered in this study. The first is partial-band noise jamming which can be caused either by a partial-band jammer or some unintended narrowband interference. If the fraction of the spread bandwidth jammed is  $\gamma$ , then the partial-band noise jamming, modeled as additive Gaussian noise and assumed to be present in both branches of the BFSK demodulator when present, affects each bit with probability  $\gamma$ . If the average power spectral density of the interference over the entire spread bandwidth is  $N_i/2$ , then the power spectral density of the partial-band interference is  $N_i/(2\gamma)$  when it is present. The second type of interference considered is wideband interference and is assumed to be caused by thermal noise or other wideband interferences. The power spectral density of this wideband noise is defined as  $N_o/2$ . Of course, when  $\gamma = 1$ , the partial-band interference is also essentially wideband in nature from the perspective of the receiver.

The power spectral density of the total noise,  $N_T/2$ , is

$$\frac{N_T}{2} = \begin{cases} \frac{N_I}{2\gamma} + \frac{N_O}{2} & \text{when jamming is present} \\ \frac{N_O}{2} & \text{when jamming is absent} \end{cases} \quad (1)$$

If the noise normalized BFSK demodulator has a noise equivalent bandwidth of  $B$  Hz, then the received noise power for a given hop  $k$  is

$$\sigma_k^2 = \begin{cases} \left( \frac{N_I}{\gamma} + N_O \right) B & \text{with probability } \gamma \\ N_O B & \text{with probability } (1-\gamma) \end{cases} \quad (2)$$

The bit rate is assumed to be  $R_b$ . With  $L$  hops/bit, the hop rate  $R_h = LR_b$ . Therefore, the noise equivalent bandwidth,  $B$ , must satisfy  $B \geq R_h$ . For this analysis,  $B$  is assumed to be equal to  $R_h$ . However, the overall spread bandwidth is assumed to be very large as compared to the hop rate. As the bit energy is assumed to be constant, the coded bit energy,  $E_c$ , is related to the uncoded bit energy,  $E_b$ , and the code rate,  $r$ , by  $E_c = r E_b$ .

It is well known that in order to maintain the same data bit rate for a coded system as for the uncoded system that the transmission bandwidth must be increased by  $1/r$ . For conventional system this may be a disadvantage, but for spread spectrum systems where the intent is to maximize transmission bandwidth, this increase is advantageous.

## II. ANALYSIS

In this thesis, an analysis of the bit error probability for the receiver illustrated in Fig. 1 is performed. This analysis requires the statistics of the sampled outputs,  $x_{ik}$ ,  $i=1,2$  of the quadrature receiver for each branch and the noise power,  $\sigma_k^2$  for each hop  $k$  of a bit.

### A. PROBABILITY OF BIT ERROR, $P_b$

It is well known that for soft-decision Viterbi decoding that the probability of bit error is upper bounded by [Ref. 4-6]

$$P_b \leq \frac{1}{k} \sum_{j=d_{\text{free}}}^{\infty} \omega_j P_j \quad (3)$$

where  $k$  is the number of input bits to the encoder,

$\omega_j$  is the total information weight of a path of weight  $j$  [Ref. 5],

$P_j$  is the probability that the all-zero path is eliminated by a path of weight  $j$ , and

$d_{\text{free}}$  is the free distance of the code.

This is the standard union bound used to evaluate the performance of coded systems. The tightness of the bound can only be established by comparison with a simulation of the system under consideration since an analytic solution does not exist. The bound is

reasonably tight for a conventional MFSK system [Ref. 5]. Consequently, it is expected that the bound will also be tight for the noise normalized receiver, especially when the number of hops per bit is small.

Since all the hops are independent and identically distributed,  $P_j$  is derived from

$$P_j = \left( \sum_{i_1=0}^L \binom{L}{i_1} \gamma^{i_1} (1-\gamma)^{L-i_1} p_1(i_1) \right) \left( \sum_{i_2=0}^L \binom{L}{i_2} \gamma^{i_2} (1-\gamma)^{L-i_2} p_2(i_2) \right) \dots \left( \sum_{i_j=0}^L \binom{L}{i_j} \gamma^{i_j} (1-\gamma)^{L-i_j} p_j(i_j) \right) \quad (4)$$

where  $p_p(i_p)$  is probability of error given that  $i_p$  of  $L$  hops in  $p^{\text{th}}$  coded transmission are jammed. This can be simplified to

$$P_j = \sum_{i_j=0}^L \sum_{i_{j-1}=0}^L \dots \sum_{i_1=0}^L \binom{L}{i_1} \binom{L}{i_2} \dots \binom{L}{i_j} \gamma^{\sum_{i=1}^j i_i} (1-\gamma)^{jL - \sum_{i=1}^j i_i} P_j(i) \quad (5)$$

where  $\underline{i} = [i_1 \ i_2 \ \dots \ i_j]$ , and  $P_j(i)$  is the conditional probability that the all-zero path is eliminated by a path of weight  $j$  given that  $\underline{i}$  hops are jammed and equals  $(p_1(i_1) \dots p_j(i_j))$ . Finally, it is possible to simplify (5) to a simple binomial expression of the form

$$P_j = \sum_{i_p=0}^{jL} \binom{jL}{\sum_{p=1}^j i_p} \gamma^{\sum_{p=1}^j i_p} (1-\gamma)^{jL - \sum_{p=1}^j i_p} P_j(i) \quad (6)$$

From [Ref. 6 : pp 34], the conditional probability  $P_j(i)$  can be calculated for binary orthogonal signaling and noncoherent detection and is equal to the probability that the sum of  $j$  samples from the signal-absent quadratic detector is greater than the sum of  $j$  samples from the signal present detector. Hence,

$$P_j(i) = 1 - \int_0^{\infty} f_{Z_1}(z_1) \left[ \int_0^{z_1} f_{Z_2}(z_2) dz_2 \right] dz_1 \quad (7)$$

where  $Z_1$  and  $Z_2$  are the random variable of the noise normalized outputs of the branches 1 and 2 respectively and is summed over  $j$  coded transmissions. It is assumed, without loss of generality, that the signal is present in branch 1.

From (7), it is observed that in order to determine the bound on bit error probabilities given by (3) for a noise normalized receiver and a Rician fading channel, we need to first determine the conditional probability density function of  $Z_1$  and the probability density function of  $Z_2$ , the noise normalized decision statistics which are formed by summing the noise normalized, sampled outputs for each hop over  $j$  coded transmissions [Ref. 4,6]. This summation of the  $j$  coded transmissions is made inside the Viterbi decoder. It should be noted, with reference to Fig. 1, that no decision is made until the differential amplifier output passes through the Viterbi decoder.

## B. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLE $Z_i$

### 1. Probability Density Function of $Z_{ik}$

To determine the probability density function of the noise normalized, sampled outputs for each hop, we let  $\sigma_k^2$  represent the noise power in a given hop  $k$  of a bit. For this thesis, we assume that  $\sigma_k^2$  is determined from the noise-only channel estimator and is known exactly. This results in an ideal performance analysis. Furthermore, without loss of generality, the signal is assumed to be in branch 1 of the BFSK demodulator. To determine the probability density function of the noise normalized random variable  $Z_{ik}$ , we start with the conditional probability density function of the random variable  $X_{ik}$  at the output of the quadratic detector. This output is equivalent to the envelope squared of a sine wave plus a narrowband process. The probability density function of the random variable used to model this signal is [Ref. 7 : pp 112, eq. 4-69]

$$p(q | \theta) = \frac{1}{2\sigma^2} \exp\left[-\frac{(q+A^2)}{2\sigma^2}\right] I_0\left[\frac{Aq^{1/2}}{\sigma^2}\right] \quad \text{for } q > 0 \quad (8)$$

where  $A$  = rms signal amplitude,

$\sigma^2$  = variance of the narrowband process, and

$I_0(\bullet)$  = modified Bessel function of the first kind and order zero.

If  $\sigma_k^2$  is the average noise power in a given hop  $k$  and the signal amplitude is  $a_i\sqrt{2}$ , the resultant conditional probability density function is

$$f_{x_{1k}}(x_{1k} | a_k) = \frac{1}{2\sigma_k^2} \exp\left[-\frac{(x_{1k} + 2a_k^2)}{2\sigma_k^2}\right] I_0\left(\frac{a_k \sqrt{2x_{1k}}}{\sigma_k^2}\right) u(x_{1k}) \quad (9)$$

where  $u(\bullet)$  is the unit step function.

Next, since the signal amplitude for each hop is modeled as a Rician random variable, then [Ref. 8 : pp. 592, eq. 7-53]

$$f_{A_k}(a_k) = \frac{a_k}{\sigma^2} \exp\left[-\frac{a_k + \alpha^2}{2\sigma^2}\right] I_0\left[\frac{a_k \alpha}{\sigma^2}\right] u(a_k) \quad (10)$$

where the total average signal power is  $a_k^2$ , which is a summation of  $\alpha^2$  {average power of the non-faded (direct) component} and  $2\sigma^2$  {average power of the Rayleigh faded (diffuse) component}. The total average signal power is assumed to remain constant from hop to hop. It should be noted that when  $\alpha^2 = 0$ , the channel is a Rayleigh fading model, and if  $2\sigma^2 = 0$ , there is no fading.

Furthermore, the noise normalized received signal is related to the sampled outputs by

$$Z_{1k} = \frac{X_{1k}}{\sigma_k^2} \quad (11)$$

In order to determine the conditional probability density function of the noise normalized random variable, we perform the linear transformation (11) to obtain



$$f_{z_{1k}}(z_{1k} | a_k) = f_{x_{1k}}(x_{1k} = z_{1k} | a_k) \frac{dx_{1k}}{dz_{1k}} \Big|_{x_{1k}=z_{1k}} \quad (12)$$

Substituting (9) into (12), we get the conditional probability density function of  $Z_{1k}$

$$f_{z_{1k}}(z_{1k} | a_k) = \frac{1}{2} \exp \left[ -\frac{\sigma_k^2 z_{1k}^2 + 2a_k^2}{2\sigma_k^2} \right] I_0 \left[ \frac{a_k \sqrt{2z_{1k}}}{\sigma_k} \right] u(z_{1k}) \quad (13)$$

The unconditional probability density function of  $Z_{1k}$  is now found from

$$f_{z_{1k}}(z_{1k}) = \int_0^\infty f_{z_{1k}}(z_{1k} | a_k) f_{A_k}(a_k) da_k \quad (14)$$

Substituting (10) and (13) into (14), we get

$$\begin{aligned} f_{z_{1k}}(z_{1k}) &= \int_0^\infty f_{z_{1k}}(z_{1k} | a_k) \underbrace{f_{A_k}(a_k)}_{\text{}} da_k \\ &= \int_0^\infty \frac{1}{2} \exp \left[ -\frac{\sigma_k^2 z_{1k}^2 + 2a_k^2}{2\sigma_k^2} \right] I_0 \left[ \frac{a_k \sqrt{2z_{1k}}}{\sigma_k} \right] \\ &\quad \frac{a_k}{\sigma_k} \exp \left[ -\frac{a_k^2 + \alpha^2}{2\sigma^2} \right] I_0 \left[ \frac{a_k \alpha}{\sigma^2} \right] da_k \end{aligned} \quad (15)$$

which can be evaluated to yield

$$\begin{aligned}
f_{z_{1k}}(z_{1k}) &= \frac{1}{2\sigma^2} \exp\left[-\left(\frac{z_{1k}}{2} + \frac{\alpha^2}{2\sigma^2}\right)\right] \int_0^\infty a_k \exp\left[-a_k^2\left(\frac{1}{\sigma_k^2} + \frac{1}{2\sigma^2}\right)\right] \\
&\quad I_0\left[\frac{a_k \sqrt{2z_{1k}}}{\sigma_k}\right] I_0\left[\frac{a_k \alpha}{\sigma^2}\right] da_k \\
&= \frac{1}{2\sigma^2} \exp\left[-\frac{z_{1k}\sigma^2 + \alpha^2}{2\sigma^2}\right] \int_0^\infty a_k \exp\left[-a_k^2\left(\frac{2\sigma^2 + \sigma_k^2}{2\sigma_k^2\sigma^2}\right)\right] \\
&\quad I_0\left[\frac{a_k \sqrt{2z_{1k}}}{\sigma_k}\right] I_0\left[\frac{a_k \alpha}{\sigma^2}\right] da_k
\end{aligned} \tag{16}$$

Here, we make the substitutions

$$\begin{aligned}
\lambda &= \frac{2\sigma^2 + \sigma_k^2}{2\sigma_k^2\sigma^2} \\
\psi &= \frac{\sqrt{2z_{1k}}}{\sigma_k} \\
\varsigma &= \frac{\alpha}{\sigma^2}
\end{aligned} \tag{17}$$

to get

$$f_{z_{1k}}(z_{1k}) = \frac{1}{2\sigma^2} \exp\left[-\frac{z_{1k}\sigma^2 + \alpha^2}{2\sigma^2}\right] \int_0^\infty a_k \exp\left[-\lambda a_k^2\right] I_0[\psi a_k] I_0[\varsigma a_k] da_k \tag{18}$$

From [Ref. 9 : pp 238, eq 4.436],

$$\int_0^{\infty} e^{-p^2 x^2} J_p(ax) J_p(bx) x \, dx = \frac{1}{2\rho^2} \exp\left[-\frac{a^2+b^2}{4\rho^2}\right] I_p\left[\frac{ab}{2\rho^2}\right] \quad (19)$$

If we let  $p = 0$  and since  $I_0(x) = J_0(ix)$ , (19) becomes

$$\int_0^{\infty} e^{-p^2 x^2} I_0(iax) I_0(ibx) x \, dx = \frac{1}{2\rho^2} \exp\left(-\frac{a^2+b^2}{4\rho^2}\right) I_0\left(\frac{ab}{2\rho^2}\right) \quad (20)$$

If we let  $\psi = ia$ ,  $\varsigma = ib$ ,  $\lambda = \rho^2$ , and  $a_k = x$ , the integral in (18) becomes

$$\begin{aligned} & \int_0^{\infty} a_k \exp\left[-\lambda a_k^2\right] I_0[\psi a_k] I_0[\varsigma a_k] \, da_k \\ &= \frac{1}{2\lambda} \exp\left[-\frac{(i\psi)^2 + (i\varsigma)^2}{4\lambda}\right] I_0\left[\frac{(i\psi)(i\varsigma)}{2\lambda}\right] \\ &= \frac{1}{2\lambda} \exp\left[\frac{\psi^2 + \varsigma^2}{4\lambda}\right] I_0\left[-\frac{\psi\varsigma}{2\lambda}\right] \\ &= \frac{1}{2\lambda} \exp\left[\frac{\psi^2 + \varsigma^2}{4\lambda}\right] I_0\left[\frac{\psi\varsigma}{2\lambda}\right] \\ &\quad \text{since } I_0(-x) = I_0(x) \end{aligned} \quad (21)$$

Substituting this into (18), we get

$$f_{z_{ik}}(z_{ik}) = \frac{1}{2\sigma^2} \exp\left[-\frac{z_{ik}\sigma^2 + \alpha^2}{2\sigma^2}\right] \frac{1}{2\lambda} \exp\left[\frac{\psi^2 + \varsigma^2}{4\lambda}\right] I_0\left[\frac{\psi\varsigma}{2\lambda}\right] \quad (22)$$

Now substituting in (17), we get

$$f_{z_{1k}}(z_{1k}) = \frac{1}{2\sigma^2} \frac{2\sigma_k^2\sigma^2}{2(2\sigma^2+2\sigma_k^2)} \exp \left[ -\frac{z_{1k}\sigma^2+\alpha^2}{2\sigma^2} + \frac{\frac{2z_{1k}\sigma^4+\alpha^2}{\sigma_k^2\sigma^4}}{4\left(\frac{2\sigma^2+\sigma_k^2}{2\sigma_k^2\sigma^2}\right)} \right] I_0 \left[ \frac{\frac{\sqrt{2z_{1k}}}{\sigma_k} \frac{\alpha}{\sigma^2}}{2\left(\frac{2\sigma^2+\sigma_k^2}{2\sigma_k^2\sigma^2}\right)} \right] \quad (23)$$

This can be simplified to

$$\begin{aligned} f_{z_{1k}}(z_{1k}) &= \frac{\sigma_k^2}{2\sigma^2+\sigma_k^2} \exp \left[ -\frac{z_{1k}\sigma^2+\alpha^2}{2\sigma^2} + \frac{(2z_{1k}\sigma^4+\alpha^2\sigma_k^2)(\sigma_k^2\sigma^3)}{(\sigma_k^2\sigma^4)[2(2\sigma^2+\sigma_k^2)]} \right] \\ &\quad I_0 \left[ \frac{\sqrt{2z_{1k}}\alpha}{2\sigma^2+\sigma_k^2} \right] \\ &= \frac{1}{2\left(\frac{2\sigma^2}{\sigma_k^2}+1\right)} \exp \left[ -\frac{z_{1k}\sigma^2+\alpha^2}{2\sigma^2} + \frac{2z_{1k}\sigma^4+\alpha^2\sigma_k^2}{2\sigma^2\sigma_k^2\left(\frac{2\sigma^2}{\sigma_k^2}+1\right)} \right] \\ &\quad I_0 \left[ \frac{\sqrt{\frac{2z_{1k}\alpha^2}{\sigma_k^2}}}{\left(\frac{2\sigma^2}{\sigma_k^2}+1\right)} \right] \end{aligned} \quad (24)$$

Next, we substitute the signal-to-noise ratio of the diffuse component,  $\xi_k = 2\sigma^2/\sigma_k^2$ , to

get

$$\begin{aligned}
f_{z_{1k}}(z_{1k}) &= \frac{1}{2(1+\xi_k)} I_0 \left[ \frac{\sqrt{\frac{2z_{1k}\alpha^2}{\sigma_k^2}}}{(1+\xi_k)} \right] \exp \left[ -\frac{z_{1k}\sigma^2 + \alpha^2}{2\sigma^2} - \frac{2z_{1k}\sigma^4 + \alpha^2\sigma_k^2}{2\sigma^2\sigma_k^2(1+\xi_k)} \right] \\
&= \frac{1}{2(1+\xi_k)} I_0 \left[ \frac{\sqrt{\frac{2z_{1k}\alpha^2}{\sigma_k^2}}}{(1+\xi_k)} \right] \exp \left[ \frac{-(z_{1k}\sigma^2 + \alpha^2)(2\sigma^2 + \sigma_k^2) + 2z_{1k}\sigma^4 + \alpha^2\sigma_k^2}{2\sigma^2\sigma_k^2(1+\xi_k)} \right]
\end{aligned} \tag{25}$$

which can be simplified to

$$\begin{aligned}
f_{z_{1k}}(z_{1k}) &= \frac{1}{2(1+\xi_k)} \exp \left[ -\frac{z_{1k}\sigma^2\sigma_k^2 + 2\alpha^2\sigma^2}{2\sigma^2\sigma_k^2(1+\xi_k)} \right] I_0 \left[ \frac{\sqrt{\frac{2z_{1k}\alpha^2}{\sigma_k^2}}}{(1+\xi_k)} \right] \\
&= \frac{1}{2(1+\xi_k)} \exp \left[ -\frac{\frac{z_{1k}}{2} + \frac{\alpha^2}{\sigma_k^2}}{1+\xi_k} \right] I_0 \left[ \frac{\sqrt{\frac{2z_{1k}\alpha^2}{\sigma_k^2}}}{(1+\xi_k)} \right]
\end{aligned} \tag{26}$$

Finally, substituting the signal-to-noise ratio of the direct component,  $\rho_k = \alpha^2 / \sigma_k^2$ , we get

$$f_{z_{1k}}(z_{1k}) = \frac{1}{2(1+\xi_k)} \exp \left[ -\frac{\frac{z_{1k}}{2} + \rho_k}{1+\xi_k} \right] I_0 \left[ \frac{\sqrt{2z_{1k}\rho_k}}{(1+\xi_k)} \right] \tag{27}$$

Therefore, the probability density function of  $Z_{1k}$  has been determined.

## 2. Probability Density Function of $Z_l$

Let  $Z_{lkp}^{(1)}$  and  $Z_{lkp}^{(2)}$  denote the random variable  $Z_{lkp}$  when hop is jammed or not jammed, respectively. For convolutional coding with soft decision Viterbi decoding, we need to determine the random variable for the sum of  $j$  coded transmissions [Ref. 9-10]. Assuming that for a coded transmission  $p$  of the  $j$  transmissions,  $i_p$  hops are jammed, we obtain the resultant decision random variable  $Z_l$

$$\begin{aligned} Z_l &= \sum_{p=1}^j \sum_{k=1}^L Z_{lkp} \\ &= \sum_{p=1}^j \sum_{k=1}^{i_p} Z_{lkp}^{(1)} + \sum_{p=1}^j \sum_{k=i_p+1}^L Z_{lkp}^{(2)} \end{aligned} \quad (28)$$

where it can be seen that  $i_p$  is not necessarily the same for each of the  $j$  coded transmissions.

Since  $Z_{lkp}^{(n)}$ ,  $n=1,2$  are independent and identically distributed random variables,

$$f_{Z_{lkl}^{(n)}}(Z_{lkl}^{(n)}) = f_{Z_{lk2}^{(n)}}(Z_{lk2}^{(n)}) = \dots = f_{Z_{lkj}^{(n)}}(Z_{lkj}^{(n)}) \quad n = 1,2 \quad (29)$$

we can simplify (28) to

$$\begin{aligned}
Z_1 &= \sum_{p=1}^j \sum_{k=1}^L Z_{1kp} \\
&= \sum_{k=1}^j i_p Z_{1k}^{(1)} + \sum_{k=\left(\sum_{p=1}^j i_p\right)+1}^{jL} Z_{1k}^{(2)} \\
&= \sum_{k=1}^{\sum_{p=1}^j i_p} Z_{1k}^{(1)} + \sum_{k=1}^{jL - \sum_{p=1}^j i_p} Z_{1k}^{(2)}
\end{aligned} \tag{30}$$

In order to determine the probability density function of  $Z_1$ , we need to find the Laplace transform of the probability density function of  $Z_{1k}^{(n)}$ . The probability density function is obtained from (27) by replacing  $\rho_k$  and  $\xi_k$  with  $\rho_k^{(n)}$  and  $\xi_k^{(n)}$ , respectively, to get

$$f_{Z_{1k}^{(n)}}(z_{1k}^{(n)}) = \frac{1}{2(1+\xi_k^{(n)})} \exp\left[-\frac{\frac{z_{1k}^{(n)}}{2} + \rho_k^{(n)}}{1+\xi_k^{(n)}}\right] I_0\left[\frac{\sqrt{2z_{1k}^{(n)}\rho_k^{(n)}}}{(1+\xi_k^{(n)})}\right] \tag{31}$$

The Laplace transform is found from

$$\begin{aligned}
F_{Z_{1k}^{(n)}}(s) &= \int_0^{\infty} f_{Z_{1k}^{(n)}}(z_{1k}^{(n)}) \exp(-sz_{1k}^{(n)}) dz_{1k}^{(n)} \\
&= \int_0^{\infty} \frac{1}{2(1+\xi_k^{(n)})} \exp\left[-\frac{1}{2}\left(\frac{z_{1k}^{(n)} + 2\rho_k^{(n)}}{1+\xi_k^{(n)}}\right)\right] I_0\left(\frac{\sqrt{2\rho_k^{(n)}z_{1k}^{(n)}}}{1+\xi_k^{(n)}}\right) \\
&\quad \exp(-sz_{1k}^{(n)}) dz_{1k}^{(n)}
\end{aligned} \tag{32}$$

This is simplified to

$$F_{Z_{1k}^{(n)}}(s) = \frac{1}{2(1+\xi_k^{(n)})} \exp\left(-\frac{\rho_k^{(n)}}{1+\xi_k^{(n)}}\right) \int_0^\infty \exp\left(-\frac{z_{1k}^{(n)}}{2(1+\xi_k^{(n)})} - sz_{1k}^{(n)}\right) I_0\left(\frac{\sqrt{2\rho_k^{(n)} z_{1k}^{(n)}}}{1+\xi_k^{(n)}}\right) dz_{1k}^{(n)} \quad (33)$$

If we substitute

$$\beta_k^{(n)} = \frac{1}{2(1+\xi_k^{(n)})} \quad (34)$$

then

$$F_{Z_{1k}^{(n)}}(s) = \beta_k^{(n)} \exp(-2\rho_k^{(n)}\beta_k^{(n)}) \int_0^\infty \exp(-z_{1k}^{(n)}(\beta_k^{(n)}+s)) I_0\left(2\beta_k^{(n)}\sqrt{2\rho_k^{(n)} z_{1k}^{(n)}}\right) dz_{1k}^{(n)} \quad (35)$$

Next, we let  $z_{1k}^{(n)} = u^2$  in (35) which implies

$$\begin{aligned} z_{1k}^{(n)} = 0 & \quad \Rightarrow \quad u = 0 \\ z_{1k}^{(n)} \rightarrow \infty & \quad \Rightarrow \quad u \rightarrow \infty \end{aligned} \quad (36)$$

Making these substitutions in (35), we get

$$F_{Z_{1k}^{(n)}}(s) = 2\beta_k^{(n)} \exp(-2\rho_k^{(n)}\beta_k^{(n)}) \int_0^\infty \exp(-u^2(\beta_k^{(n)}+s)) I_0\left(2\beta_k^{(n)}u\sqrt{2\rho_k^{(n)}}\right) u \, du \quad (37)$$



From [Ref. 10 : pp 486, eq 11.4.29],

$$\int_0^{\infty} e^{-a^2 t^2} t^{\nu+1} J_{\nu}(bt) dt = \frac{b^{\nu}}{(2a^2)^{\nu+1}} e^{-b^2/4a^2} \quad (38)$$

If we let  $\nu = 0$ , and since  $I_0 = J_0(ix)$ , (38) becomes

$$\int_0^{\infty} e^{-a^2 t^2} t I_0(ibt) dt = \frac{1}{2a^2} e^{-b^2/4a^2} \quad (39)$$

In addition, we let

$$\begin{aligned} a^2 &= \beta_k^{(n)} + s \\ b &= 2i\beta_k^{(n)} \sqrt{2\rho_k^{(n)}} \end{aligned} \quad (40)$$

Substituting (39) and (40) into the integral portion of (37), we get

$$\begin{aligned}
& \int_0^{\infty} \exp(-u^2(\beta_k^{(n)}+s)) I_0(2\beta_k^{(n)}\sqrt{2\rho_k^{(n)}} u) u \, du \\
&= \int_0^{\infty} \exp(-u^2 a^2) I_0(ibu) u \, du \\
&= \frac{1}{2a^2} e^{-b^2/4a^2} \\
&= \frac{1}{2(\beta_k^{(n)}+s)} \exp\left(\frac{8(\beta_k^{(n)})^2 \rho_k^{(n)}}{4(\beta_k^{(n)}+s)}\right) \\
&= \frac{1}{2(\beta_k^{(n)}+s)} \exp\left(\frac{2(\beta_k^{(n)})^2 \rho_k^{(n)}}{\beta_k^{(n)}+s}\right)
\end{aligned} \tag{41}$$

Putting (41) into (37), we get

$$\begin{aligned}
F_{Z_{ik}}(s) &= \frac{\beta_k^{(n)}}{\beta_k^{(n)}+s} \exp\left(-2\rho_k^{(n)}\beta_k^{(n)} + \frac{2(\beta_k^{(n)})^2 \rho_k^{(n)}}{\beta_k^{(n)}+s}\right) \\
&= \frac{\beta_k^{(n)}}{\beta_k^{(n)}+s} \exp\left(\frac{-2\rho_k^{(n)}\beta_k^{(n)}(\beta_k^{(n)}+s) + 2(\beta_k^{(n)})^2 \rho_k^{(n)}}{\beta_k^{(n)}+s}\right) \\
&= \frac{\beta_k^{(n)}}{\beta_k^{(n)}+s} \exp\left(-\frac{2\rho_k^{(n)}\beta_k^{(n)}s}{\beta_k^{(n)}+s}\right)
\end{aligned} \tag{42}$$

Since all the hops are independent, we obtain the conditional probability density function for the decision variable  $Z_i$  given that  $i_1, i_2, \dots, i_j$  hops of the  $j$  bits have interference as

$$f_{Z_1}(z_1 \mid i_1, i_2, \dots, i_j) = \left[ \left( f_{Z_{1k}^{(1)}}(z_{1k}^{(1)}) \right)^{\otimes i_1} \otimes \left( f_{Z_{1k}^{(2)}}(z_{1k}^{(2)}) \right)^{\otimes (L-i_1)} \right] \otimes \left[ \left( f_{Z_{1k}^{(1)}}(z_{1k}^{(1)}) \right)^{\otimes i_2} \otimes \left( f_{Z_{1k}^{(2)}}(z_{1k}^{(2)}) \right)^{\otimes (L-i_2)} \right] \dots \otimes \left[ \left( f_{Z_{1k}^{(1)}}(z_{1k}^{(1)}) \right)^{\otimes i_j} \otimes \left( f_{Z_{1k}^{(2)}}(z_{1k}^{(2)}) \right)^{\otimes (L-i_j)} \right] \quad (43)$$

where  $a \otimes b$  implies the convolution of  $a$  and  $b$ , and  $a^{\otimes c}$  represents a  $c$ -fold convolution of  $a$ . Equation (43) simplifies to

$$\begin{aligned} f_{Z_1}(z_1 \mid i_1, i_2, \dots, i_j) &= \left( f_{Z_{1k}^{(1)}}(z_{1k}^{(1)}) \right)^{\otimes (i_1 + i_2 + \dots + i_j)} \\ &\quad \otimes \left( f_{Z_{1k}^{(2)}}(z_{1k}^{(2)}) \right)^{\otimes (L-i_1 + L-i_2 + \dots + L-i_j)} \\ &= \left( f_{Z_{1k}^{(1)}}(z_{1k}^{(1)}) \right)^{\otimes \sum_{p=1}^j i_p} \otimes \left( f_{Z_{1k}^{(2)}}(z_{1k}^{(2)}) \right)^{\otimes \left[ jL - \sum_{p=1}^j i_p \right]} \end{aligned} \quad (44)$$

In order to determine the convolution, we will multiply in the  $s$  domain. That is, from (42) we get the Laplace transform of the conditional probability density function of  $Z_1$  given that  $i_1, i_2, \dots, i_j$  hops have interference

$$F_{Z_1}(s \mid \mathbf{i}) = \left[ \frac{\beta_k^{(1)}}{\beta_k^{(1)} + s} \exp \left( - \frac{2\rho_k^{(1)}\beta_k^{(1)}s}{\beta_k^{(1)} + s} \right) \right]^{\sum_{p=0}^j i_p} \left[ \frac{\beta_k^{(2)}}{\beta_k^{(2)} + s} \exp \left( - \frac{2\rho_k^{(2)}\beta_k^{(2)}s}{\beta_k^{(2)} + s} \right) \right]^{jL - \sum_{p=0}^j i_p} \quad (45)$$

The above Laplace transform cannot be inverted analytically except when either all hops are jammed or no hops are jammed. In these two cases, the inverse Laplace transform of (45) is

$$\begin{aligned}
\mathcal{L}^{-1}(\mathbf{F}_{z_1}(s)) &= \mathcal{L}^{-1}[(\mathbf{F}_{z_{ik}^{(n)}}(s))^{\mathbf{jL}}] \\
&= (\beta_k^{(n)})^{\mathbf{jL}} \exp(-2\mathbf{jL}\beta_k^{(n)}\rho_k^{(n)}) \\
&\quad \mathcal{L}^{-1}\left[\left(\frac{1}{s+\beta_k^{(n)}}\right)^{\mathbf{jL}} \exp\left(\frac{2\mathbf{jL}(\beta_k^{(n)})^2\rho_k^{(n)}}{s+\beta_k^{(n)}}\right)\right] \\
&= (\beta_k^{(n)})^{\mathbf{jL}} \exp(-2\mathbf{jL}\beta_k^{(n)}\rho_k^{(n)}) \exp(-\beta_k^{(n)}z_1) \\
&\quad \mathcal{L}^{-1}\left[\left(\frac{1}{s}\right)^{\mathbf{jL}} \exp\left(\frac{2\mathbf{jL}(\beta_k^{(n)})^2\rho_k^{(n)}}{s}\right)\right]
\end{aligned} \tag{46}$$

From [Ref. 11 : pp 59, eq 81], we get

$$\mathcal{L}^{-1}\left(\frac{1}{s^\mu} e^{k/s}\right) = \left(\frac{t}{k}\right)^{(\mu-1)/2} I_{\mu-1}(2\sqrt{kt}) \tag{47}$$

Let

$$\begin{aligned}
k &= 2\mathbf{jL}(\beta_k^{(n)})^2 \rho_k^{(n)} \\
\mu &= \mathbf{jL}
\end{aligned} \tag{48}$$

Substituting this into the inverse Laplace transform on the right hand side of (46), we get

$$\begin{aligned}
\mathcal{L}^{-1}\left[\left(\frac{1}{s}\right)^{\mathbf{jL}} \exp\left(\frac{2\mathbf{jL}(\beta_k^{(n)})^2\rho_k^{(n)}}{s}\right)\right] &= \left(\frac{z_1}{2\mathbf{jL}(\beta_k^{(n)})^2\rho_k^{(n)}}\right)^{\frac{\mathbf{jL}-1}{2}} \\
&\quad I_{\mathbf{jL}-1}\left(2\sqrt{2\mathbf{jL}(\beta_k^{(n)})^2\rho_k^{(n)}}z_1\right)
\end{aligned} \tag{49}$$

Therefore, substituting (49) into (46), we get the probability density function of  $Z_i$  for either no jamming or full band jamming as

$$\begin{aligned}
 f_{Z_i}(z_1) &= \mathcal{F}^{-1} \left[ \left( F_{Z_{ik}^{(n)}}(s) \right)^{jL} \right] \\
 &= (\beta_k^{(n)})^{jL} \exp \left( -2jL\beta_k^{(n)}\rho_k^{(n)} - \beta_k^{(n)}z_1 \right) \\
 &\quad \left( \frac{z_1}{2jL(\beta_k^{(n)})^2\rho_k^{(n)}} \right)^{\frac{jL-1}{2}} I_{jL-1} \left( 2\sqrt{2jL(\beta_k^{(n)})^2\rho_k^{(n)}z_1} \right)
 \end{aligned} \tag{50}$$

$$\begin{aligned}
 f_{Z_i}(z_1) &= (\beta_k^{(n)})^{jL} \exp \left[ -\beta_k^{(n)} \left( z_1 + 2jL\rho_k^{(n)} \right) \right] (z_1)^{\frac{jL-1}{2}} \\
 &\quad \left( \frac{1}{2jL\rho_k^{(n)}} \right)^{\frac{jL-1}{2}} \frac{1}{(\beta_k^{(n)})^{jL-1}} I_{jL-1} \left( 2\beta_k^{(n)} \sqrt{2jL\rho_k^{(n)}z_1} \right)
 \end{aligned} \tag{51}$$

This can be simplified to

$$\begin{aligned}
 f_{Z_i}(z_1) &= \frac{\beta_k^{(n)}(z_1)^{\frac{jL-1}{2}}}{(2jL\rho_k^{(n)})^{\frac{jL-1}{2}}} \exp \left[ -\beta_k^{(n)} \left( z_1 + 2jL\rho_k^{(n)} \right) \right] \\
 &\quad I_{jL-1} \left( 2\beta_k^{(n)} \sqrt{2jL\rho_k^{(n)}z_1} \right)
 \end{aligned} \tag{52}$$

Therefore, the probability density function of  $Z_i$  has been derived.

### 3. Probability Density Function of $Z_i$ with Rayleigh Fading

For the special case of Rayleigh fading,  $\rho_k \rightarrow 0$ , and the Laplace transform of the conditional probability density function of  $Z_i$  given  $i_1, i_2, \dots, i_j$  given by (46) reduces to

$$F_{Z_i}(s \mid \mathbf{i}) = \left[ \frac{\beta_k^{(1)}}{\beta_k^{(1)} + s} \right]^{\sum_{p=0}^j i_p} \left[ \frac{\beta_k^{(2)}}{\beta_k^{(2)} + s} \right]^{jL - \sum_{p=0}^j i_p}. \quad (53)$$

Depending on the values of  $i_p$ , the powers of the bracketed terms can theoretically vary up to infinity. Furthermore, we know that  $\beta_k^{(n)}$  is a constant for each hop and each bit. Therefore, we let

$$\begin{aligned} m &= \sum_{p=1}^j i_p \\ n &= jL - \sum_{p=1}^j i_p \\ a &= \beta_k^{(1)} \\ b &= \beta_k^{(2)} \end{aligned} \quad (54)$$

and (53) simplifies to

$$\begin{aligned} F_{Z_i}(s \mid \mathbf{i}) &= \left( \frac{a}{s+a} \right)^m \left( \frac{b}{s+b} \right)^n \\ &= a^m b^n \left( \frac{1}{(s+a)^m} \frac{1}{(s+b)^n} \right) \end{aligned} \quad (55)$$

By using partial fractions for the above equation, we get

$$F_{Z_1}(s | i) = a^m b^n \left[ \frac{K_{11}}{(s+a)} + \frac{K_{12}}{(s+a)^2} + \frac{K_{13}}{(s+a)^3} + \dots + \frac{K_{1m}}{(s+a)^m} + \frac{K_{21}}{(s+b)} + \frac{K_{22}}{(s+b)^2} + \frac{K_{23}}{(s+b)^3} + \dots + \frac{K_{2n}}{(s+b)^n} \right] \quad (56)$$

Using Heaviside's Expansion Theorem for partial fractions [Ref. 12 : pp 174, eq 7-83],

we get

$$K_{jx} = \frac{1}{(r-x)!} \frac{d^{r-x}}{ds^{r-x}} R_j(s) \Big|_{s=a_j-b} \quad (57)$$

where  $j = 1, 2$

$r$  = highest power of the denominators (i.e., if  $j=1$ ,  $r=m$ ; and if  $j=2$ ,  $r=n$ )

$x$  = the  $x^{\text{th}}$  term of the expansion

and

$$\begin{aligned} R_1(s) &= \frac{1}{(s+b)^n} \\ R_2(s) &= \frac{1}{(s+a)^m} \end{aligned} \quad (58)$$

Therefore,

$$\begin{aligned} K_{1m}(s) &= R_1(s) \Big|_{s=-a} = \frac{1}{(b-a)^n} \\ K_{2n}(s) &= R_2(s) \Big|_{s=-b} = \frac{1}{(a-b)^m} \end{aligned} \quad (59)$$

Furthermore,

$$\begin{aligned}
K_{1(m-1)} &= \frac{1}{(m-m+1)!} \left. \frac{d}{ds} R_1(s) \right|_{s=-a} \\
&= -\frac{n}{(s+b)^{n+1}} \Big|_{s=-a} \\
&= -\frac{n}{(b-a)^{n+1}}
\end{aligned} \tag{60}$$

$$\begin{aligned}
K_{1(m-2)} &= \frac{1}{2!} \left. \frac{d^2}{ds^2} R_1(s) \right|_{s=-a} \\
&= \frac{1}{2!} \left( \frac{n(n+1)}{(b-a)^{n+2}} \right)
\end{aligned} \tag{61}$$

$$\begin{aligned}
K_{1(m-i)} &= \frac{1}{i!} \left. \frac{d^i}{ds^i} R_1(s) \right|_{s=-a} \\
&= \frac{1}{i!} \left( \frac{(-1)^i n(n+1)(n+2) \dots (n+i-1)}{(b-a)^{n+i}} \right)
\end{aligned} \tag{62}$$

$$K_{11} = \frac{1}{(m-1)!} \left( \frac{(-1)^{m-1} n(n+1)(n+2) \dots (n+m-2)}{(b-a)^{n+m-1}} \right) \tag{63}$$

Similarly,

$$\begin{aligned}
K_{2(n-1)} &= \frac{1}{(n-n+1)!} \left. \frac{d}{ds} R_2(s) \right|_{s=-b} \\
&= -\frac{m}{(s+a)^{m+1}} \Big|_{s=-b} \\
&= -\frac{m}{(a-b)^{m+1}}
\end{aligned} \tag{64}$$



$$\begin{aligned}
K_{2(n-2)} &= \frac{1}{2!} \left. \frac{d^2}{ds^2} R_2(s) \right|_{s=a-b} \\
&= \frac{1}{2!} \left( \frac{m(m+1)}{(a-b)^{m+2}} \right)
\end{aligned} \tag{65}$$

$$\begin{aligned}
K_{2(n-i)} &= \frac{1}{i!} \left. \frac{d^i}{ds^i} R_2(s) \right|_{s=a-b} \\
&= \frac{1}{i!} \left( \frac{(-1)^i m(m+1)(m+2) \dots (m+i-1)}{(a-b)^{m+i}} \right)
\end{aligned} \tag{66}$$

$$K_{21} = \frac{1}{(n-1)!} \left( \frac{(-1)^{n-1} m(m+1)(m+2) \dots (n+m-2)}{(a-b)^{n+m-1}} \right) \tag{67}$$

Using the following Laplace transform pairs,

$$\begin{aligned}
f(t) = t^n &\leftrightarrow F(s) = \frac{n!}{s^{n+1}} \\
f(t) = e^{-at} &\leftrightarrow F(s) = \frac{1}{s+a}
\end{aligned} \tag{68}$$

we get,

$$\begin{aligned}
\mathcal{L}^{-1} \left( \frac{a^m}{(s+a)^m} \right) &= a^m e^{-at} \frac{t^{m-1}}{(m-1)!} \\
\mathcal{L}^{-1} \left( \frac{b^n}{(s+b)^n} \right) &= b^n e^{-bt} \frac{t^{n-1}}{(n-1)!}
\end{aligned} \tag{69}$$

Therefore, the inverse Laplace transform (53) is found to be

$$f_{z_1}(z_1|i) = \left[ K_{11} + K_{12}z_1 + \dots + K_{1m} \frac{z_1^{m-1}}{(m-1)!} \right] \exp(-\beta_{kl}^{(1)} z_1) u(z_1) + \left[ K_{21} + K_{22}z_1 + \dots + K_{2n} \frac{z_1^{n-1}}{(n-1)!} \right] \exp(-\beta_{kl}^{(2)} z_2) u(z_1) \quad (70)$$

Using  $m=1$  and  $n=2$  in (70), we get

$$\begin{aligned} K_{11} &= \frac{1}{(b-a)^2} \\ K_{22} &= \frac{1}{(a-b)} \\ K_{21} &= -\frac{1}{(a-b)^2} \end{aligned} \quad (71)$$

Next, using  $m=2$  and  $n=2$  in (70), we get

$$\begin{aligned} K_{12} &= \frac{1}{(b-a)^2} \\ K_{11} &= -\frac{2}{(b-a)^3} \\ K_{22} &= \frac{1}{(a-b)^2} \\ K_{21} &= -\frac{2}{(a-b)^3} \end{aligned} \quad (72)$$

It can be seen from the above that the results are as expected in [Ref. 1 : eq 15].

### C. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLE $Z_2$

In order to get the probability density function of  $Z_{2k}$ , we let the direct ( $\rho_k$ ) and diffuse ( $\xi_k$ ) components be zero and  $\beta_k^{(n)} = 1/2$  in (42). This result is then raised to the  $(jL)^{\text{th}}$  power to yield

$$\begin{aligned} F_{Z_2}(s) &= \left( \frac{\beta_k^{(n)}}{\beta_k^{(n)} + s} \right)^{jL} \\ &= \left( \frac{1}{2(s + \frac{1}{2})} \right)^{jL} \end{aligned} \quad (73)$$

which is inverted easily to obtain

$$f_{Z_2}(z_2) = \frac{(z_2/2)^{jL-1}}{2(jL-1)!} \exp\left(-\frac{z_2}{2}\right) \quad (74)$$

The dramatic simplification in obtaining the probability density function for  $Z_2$  occurs due to the normalization which makes the output of the branch that does not contain the signal independent of whether the hop is jammed or not. Next, we need to find

$$\int_0^{z_1} f_{Z_2}(z_2) dz_2 = \int_0^{z_1} \frac{(z_2/2)^{jL-1}}{2(jL-1)!} \exp\left(-\frac{z_2}{2}\right) dz_2 \quad (75)$$

If we let  $x = z_2/2$ , this gives

$$\begin{aligned}
 dx &= \frac{dz_2}{2} \\
 z_2 = 0 &\Rightarrow x = 0 \\
 z_2 = z_1 &\Rightarrow x = \frac{z_1}{2}
 \end{aligned}
 \tag{76}$$

Substituting the above into (75), we obtain the simplified integral

$$\begin{aligned}
 \int_0^{z_1} f_{z_2}(z_2) dz_2 &= \int_0^{z_1} \frac{(z_2/2)^{jL-1}}{2(jL-1)!} e^{-\frac{z_2}{2}} dz_2 \\
 &= \int_0^{z_1/2} \frac{x^{jL-1}}{2(jL-1)!} e^{-x} 2 dx \\
 &= \frac{1}{(jL-1)!} \int_0^{z_1/2} x^{jL-1} e^{-x} dx
 \end{aligned}
 \tag{77}$$

From [Ref. 9 : pp 89, eq 2.321.2], we get

$$\int x^n e^{ax} dx = e^{ax} \left( \frac{x^n}{a} + \sum_{k=1}^n \frac{n(n-1)\dots(n-k+1)}{a^{k+1}} x^{n-k} \right)
 \tag{78}$$

If we let  $n = (jL-1)$  and  $a = -1$  in (78) and use this result in (77), we get

$$\begin{aligned}
\int_0^{z_1} f_{z_2}(z_2) dz_2 &= \frac{1}{(jL-1)!} \int_0^{z_1/2} x^{jL-1} e^{-x} dx \\
&= \left[ \frac{e^{-x}}{(jL-1)!} \left( -x^{jL-1} \right. \right. \\
&\quad \left. \left. + \sum_{k=1}^{jL-1} (-1)^k \frac{(jL-1) \dots (jL-k)}{(-1)^k} x^{jL-k-1} \right) \right]_{x=0}^{x=z_1/2} \\
&= \left[ \frac{e^{-x}}{(jL-1)!} \left( -x^{jL-1} - \sum_{k=1}^{jL-1} (jL-1) \dots (jL-k) x^{jL-k-1} \right) \right]_{x=0}^{x=z_1/2}
\end{aligned} \tag{79}$$

If we substitute the limits of the integral, we get

$$\begin{aligned}
\int_0^{z_1} f_{z_2}(z_2) dz_2 &= \frac{e^{-z_1/2}}{(jL-1)!} \left[ -\left(\frac{z_1}{2}\right)^{jL-1} \right. \\
&\quad \left. - \sum_{k=1}^{jL-1} (jL-1) \dots (jL-k) \left(\frac{z_1}{2}\right)^{jL-1-k} \right] + \frac{(jL-1)!}{(jL-1)!}
\end{aligned} \tag{80}$$

This can be simplified to obtain

$$\begin{aligned}
\int_0^{z_1} f_{Z_2}(z_2) \, dz_2 &= 1 - \left[ \frac{e^{z_1/2}}{(jL-1)!} \left( \frac{z_1}{2} \right)^{jL-1} \right. \\
&\quad \left. + e^{-z_1/2} \sum_{k=1}^{jL-1} \frac{(jL-1) \dots (jL-k)}{(jL-1)!} \left( \frac{z_1}{2} \right)^{jL-1-k} \right] \\
&= 1 - \left[ \frac{e^{-z_1/2}}{(jL-1)!} \left( \frac{z_1}{2} \right)^{jL-1} \right. \\
&\quad \left. + e^{-z_1/2} \sum_{k=1}^{jL-1} \frac{(z_1/2)^{jL-1-k}}{(jL-1-k)!} \right]
\end{aligned} \tag{81}$$

Therefore, the integral of the probability density function for  $Z_2$  is

$$\int_0^{z_1} f_{Z_2}(z_2) \, dz_2 = 1 - \left[ e^{-z_1/2} \sum_{k=0}^{jL-1} \frac{(z_1/2)^{jL-1-k}}{(jL-1-k)!} \right] \tag{82}$$

### III. NUMERICAL RESULTS

#### A. NUMERICAL PROCEDURE

The computation of the bit error probability requires numerical computation of (7) with the probability density function of  $Z_1$  being computed by finding the inverse Laplace transform of (45) numerically except for the two special cases of either all hops jammed or no hops jammed. The remainder of the integrand of (7) is given by (82). The results of this numerical computation are then substituted into (5).

This result for  $P_j$  is then substituted into (3) where the weights ( $\mathbb{W}_j$ ), the coding free distance ( $d_{free}$ ), and the number of input bits ( $k$ ) are used to determine the probability of bit error.

System performance is evaluated for various values of jamming fraction  $\gamma$ , diversity, fading conditions, and values of the bit energy-to-jamming noise density ratio ( $E_b/N_j$ ). For the Rician case, the ratio of direct-to-diffuse signal energy ( $\alpha^2/2\sigma^2$ ) is assumed to be the same for each hop  $k$  of the bit.

In this study,  $E_b/N_0 = 20\text{dB}$  is used when  $\alpha^2/2\sigma^2 \leq 1$ . Otherwise,  $E_b/N_0 = 13.35\text{dB}$  is used. It is expected that when  $E_b/N_0$  is reduced, performance will improve at lower values of  $E_b/N_j$  since  $E_b/N_0$  is more dominant. However, the improvement is inconsequential since for low  $E_b/N_j$  system performance with convolutional coding is too poor to be useful. It seems reasonable to assume that the trends obtained with these values of  $E_b/N_0$  will not be significantly affected as  $E_b/N_0$  increases.

## B. PERFORMANCE WITH RAYLEIGH FADING

Figures 2 to 10 shows the performance of the noise normalized receiver with Rayleigh fading. It is observed from these figures that coding improves the performance when  $E_b/N_t$  is greater than 10dB.

Figures 2 and 3 shows the performance of the receiver when  $\gamma = 1.0$  (full band jamming), 0.25, 0.1 and 0.01 and the number of hops per bit  $L = 1$  with constraint length  $\nu = 2$  and 4, respectively. The figures show that the asymptotic probability of bit errors vary from about  $10^{-6}$  ( $\nu = 2$ ) to about  $10^{-8}$  ( $\nu = 4$ ). These asymptotic probability of bit errors are much better than that obtained for uncoded performance ( $10^{-2}$ ). Comparing Figs. 2 and 3 with Figs. 4 and 5, where  $L$  is increased to 2, performance is seen to have improved with the asymptotic bit error probability being  $10^{-9}$  ( $\nu = 2$ ) and  $10^{-12}$  ( $\nu = 4$ ) as compared to  $10^{-6}$  and  $10^{-8}$  when  $L = 1$ . It is also seen that for Rayleigh fading the worst case jamming strategy to be used is full band jamming.

By increasing  $\nu$ , performance is further improved. This improvement is much more dramatic at high  $E_b/N_t$  as can be seen in Fig. 6. Comparing Fig. 6 with Fig. 7, it is observed that the asymptotic bit error probability is improved either by increasing  $\nu$  or  $L$ . The asymptotic bit error probabilities for various values of  $\nu$  and  $L$  are given in Table 1.



TABLE 1

Constraint Length, $\nu$	Asymptotic Probability of Bit Error	
	$L = 1$	$L = 2$
$\nu = 2$	$10^{-6}$	$10^{-9}$
$\nu = 4$	$10^{-8}$	$10^{-12}$
$\nu = 6$	$10^{-11}$	$< 10^{-13}$

It can also be seen in Fig. 8 that when  $E_b/N_t$  is small ( $< 15\text{dB}$ ) and  $L$  is increased that the non-coherent combining losses are dominant and degrade the performance. However, for higher  $E_b/N_t$ , the diversity gain is dominant and improves the performance of the receiver.

In Figs. 9 and 10, soft-decision Viterbi decoding is compared to hard-decision Viterbi decoding (for  $\nu = 6$ ). It is observed that soft decision decoding improves performance over hard decision decoding by about 7 to 10dB when  $E_b/N_t$  is around 10 to 20dB (Fig. 9). The asymptotic bit error probability for rate = 1/2 is seen to be  $10^{-11}$  as compared to  $10^{-4}$ . Furthermore, for rate = 1/3, the asymptotic bit error probability is  $10^{-14}$  as compared to  $10^{-6}$ . As expected, there is further improvement when  $L$  is increased (Fig. 10).

### C. PERFORMANCE WITH RICIAN FADING

Figures 11 to 17, Figs. 18 to 24, and Figs. 25 to 26 show the performance of the receiver for a strong direct signal ( $\alpha^2/2\sigma^2 = 10$ ), a weak direct signal ( $\alpha^2/2\sigma^2 = 1$ ), and a very strong direct signal ( $\alpha^2/2\sigma^2 = 100$ ), respectively. It is observed that coding improves performance when  $E_b/N_j$  is greater than about 10dB.

Figures 11 and 12 show the performance of the receiver when  $\gamma = 1.0$  (full band jamming), 0.25, 0.1 and 0.01 and  $\nu = 2$  for  $L = 1$  and 2, respectively. The asymptotic bit error probability is about  $10^{-6}$  as compared to  $10^{-2}$  for the uncoded case. The comparable situation, but with a weak direct signal component ( $\alpha^2/2\sigma^2 = 1$ ), is shown in Figs. 18 and 19. In this case, performance is slightly worse as compared to the case with  $\alpha^2/2\sigma^2 = 10$ . It is observed from all the above cases that the worst case jamming strategy is quite close to full band jamming, even when  $L = 1$ , except when  $\alpha^2/2\sigma^2 = 100$  (Figs. 25 and 26). Even then, partial-band jamming does not significantly degrade performance. Therefore, it is observed that convolutional coding improves the performance of frequency-hopping systems by forcing the jammer to spread its resources over a much wider band as compared to the non-coded system. This conclusion remains valid even when  $L=1$ . It should also be noted that with a very strong direct signal, the effect of the jammer is much less significant than when the direct signal is weaker for the same  $E_b/N_j$ .

By increasing  $\nu$  (Figs. 13, 14, 20, and 21), it is observed that the performance is improved by about 1dB as  $\nu$  is increased from 2 to 4 and from 4 to 6. This improvement

reduces the asymptotic bit error probability for strong direct signal from about  $10^{-5}$  to about  $10^{-8}$  and for the weak direct signal from about  $10^{-4}$  to  $10^{-7}$  when  $\nu = 6$ .

Figure 15 shows the performance of the receiver when there is an increase in diversity. It is observed that for the strong direct signal ( $\alpha^2/2\sigma^2 = 10$ ) that the non-coherent combining loss is greater than the diversity gain since performance is degraded with an increase in diversity. For the weak direct signal case ( $\alpha^2/2\sigma^2 = 1$ ) (Fig. 22), diversity gain is greater than the non-coherent combining losses when  $E_b/N_i$  is greater than 15dB.

With  $\nu = 6$  (Figs. 16, 17, 23 and 24), when the rate is reduced from 1/2 to 1/3, it is observed that there is minimal increase in performance when there is a strong direct signal. When there is a weak direct signal, the improvement in performance is about 0.5 to 1.0dB. Comparing these to hard decision decoding, there is an improvement of about 5dB.

From the above discussions, it is concluded that with the implementation of convolutional coding with soft decision Viterbi decoding that performance is generally improved when  $E_b/N_i$  is higher than about 10dB. Another conclusion is that convolutional coding and soft decision Viterbi decoding render fast frequency-hopping with  $L > 1$  largely unnecessary.

#### IV. CONCLUSION

When convolutional coding with soft decision Viterbi decoding is employed in conjunction with FFH/BFSK utilizing noise normalization combining, system performance is superior to the performance of the equivalent uncoded system when  $E_b/N_f$  is greater than 10dB. Below 10dB the uncoded system performance is too poor for coding to be effective. This improvement in performance is particularly true for Rayleigh fading channels. In addition, at high  $E_b/N_f$ , the asymptotic probability of bit error improves dramatically as compared to the uncoded system with the probability of bit error varying from  $10^{-6}$  to  $10^{-12}$  depending on  $\nu$ ,  $L$ , and  $(\alpha^2/2\sigma^2)$  as compared to about  $10^{-3}$  with no coding. The jamming strategy required to cause worst case receiver performance is full band jamming, even when  $L=1$ , except for very strong direct signal ( $\alpha^2/2\sigma^2 = 100$ ). Even then, partial-band jamming does not significantly degrade performance. This is a significant departure from what is observed for the uncoded system. This means that the enemy jammer must spread its jamming power over a much wider bandwidth. It also means that fast frequency-hopping with  $L>1$  is largely unnecessary when convolutional coding and soft decision Viterbi decoding are used.

Due to non-coherent combining losses, there is some degradation in performance when the hop per bit ratio is increased and the received signal is at moderate  $E_b/N_f$ .

It is found, as expected, that when a stronger code (higher constraint length) is used that performance is improved, especially for high  $E_b/N_f$ . Finally, soft decision decoding

improves the performance over the hard decision decoding by about 4 to 8 dB (depending on the code rate) at moderate  $E_b/N_f$  and has a much lower asymptotic error for high  $E_b/N_f$ .

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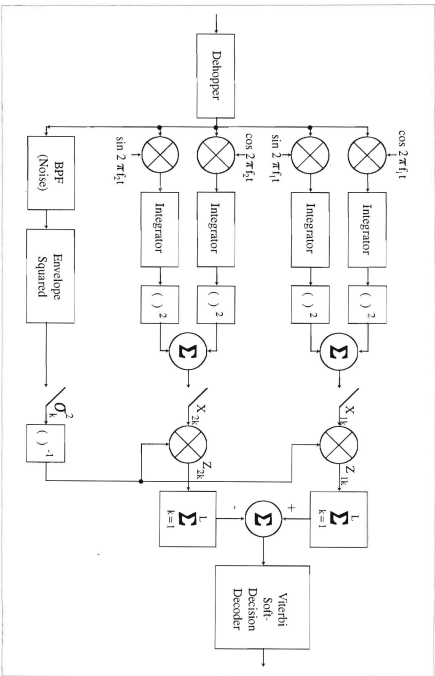


Fig. 1 : FFH/BFSK Receiver with Noise Normalization and Soft-Decision Viterbi Decoding

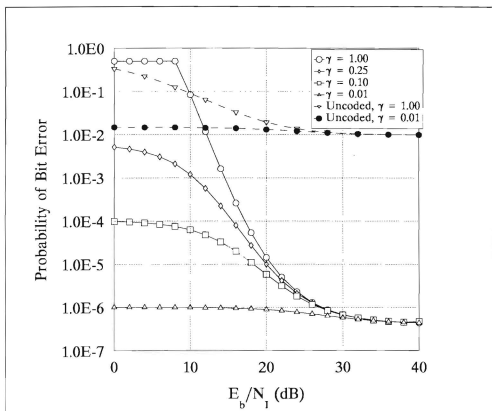


Fig. 2 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rayleigh fading and  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 1$ , and  $v = 2$ .



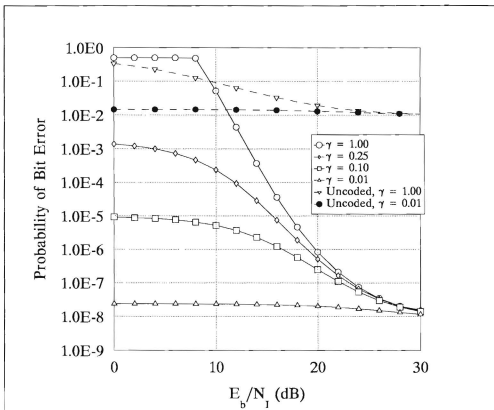


Fig. 3 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rayleigh fading, and  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 1$  and  $v = 4$ .

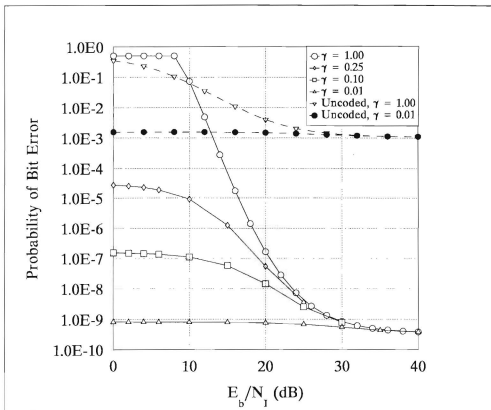


Fig. 4 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rayleigh fading, and  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 2$  and  $v = 2$ .

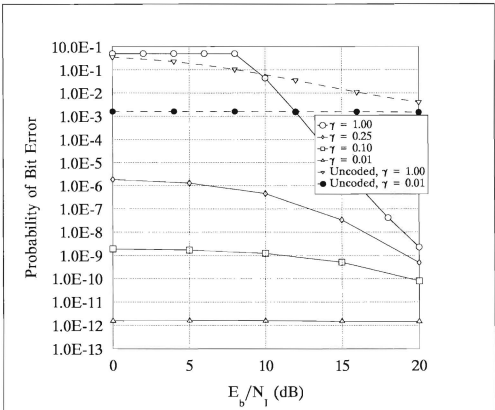


Fig. 5 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rayleigh fading, and  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 2$  and  $v = 4$ .

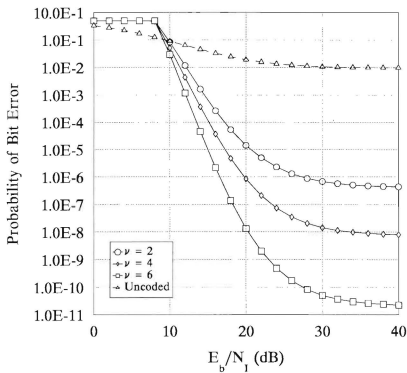


Fig. 6 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\nu = 2, 4$  and  $6$  with Rayleigh fading, full band jamming,  $E_b/N_0 = 20.0\text{dB}$ , and  $L = 1$ .

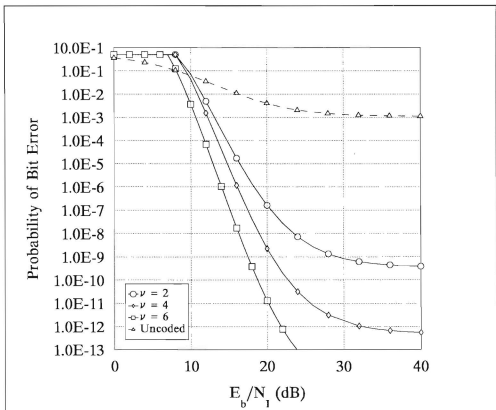


Fig. 7 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\nu = 2, 4$  and  $6$  with Rayleigh fading, full band jamming,  $E_c/N_0 = 20.0\text{dB}$ , and  $L = 2$ .

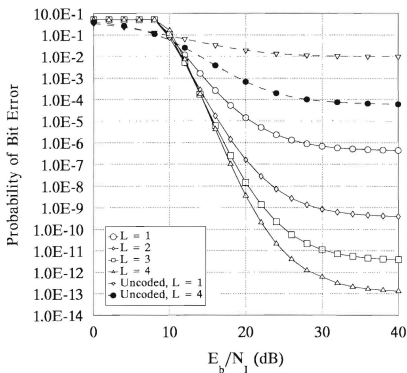


Fig. 8 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for number of hops/bit of  $L = 1, 2, 3$  and  $4$  with Rayleigh fading, full band jamming,  $E_b/N_0 = 20.0\text{dB}$ , and  $v = 2$ .

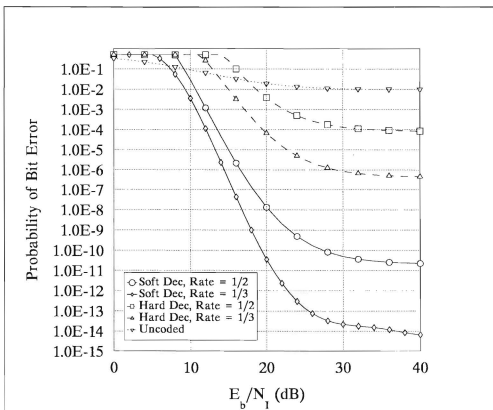


Fig. 9 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate,  $r = 1/2$  and  $1/3$  with Rayleigh fading, full band jamming,  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 1$ , and  $\nu = 6$ .

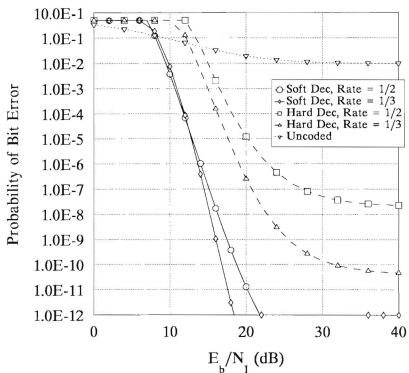


Fig. 10 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate,  $r = 1/2$  and  $1/3$  with Rayleigh fading, full band jamming,  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 2$ , and  $v = 6$ .



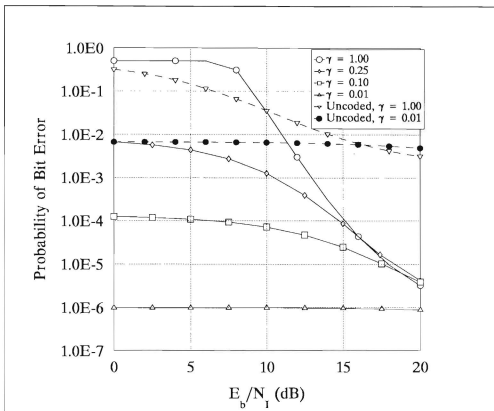


Fig. 11 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rician fading, a strong direct signal ( $\alpha^2/2\sigma^2 = 10$ ),  $E_b/N_0 = 13.35\text{dB}$ ,  $L = 1$ , and  $\nu = 2$ .

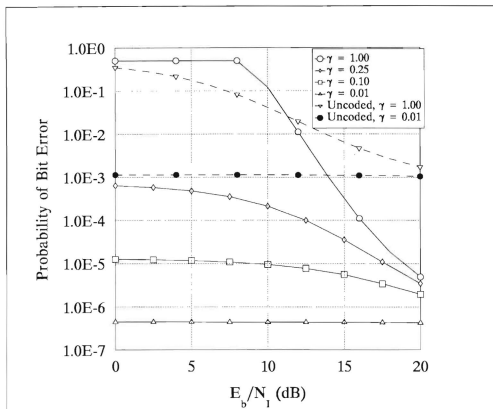


Fig. 12 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rician fading, a strong direct signal ( $\sigma^2/2\sigma^2 = 10$ ),  $E_b/N_0 = 13.35\text{dB}$ ,  $L = 2$ , and  $\nu = 2$ .

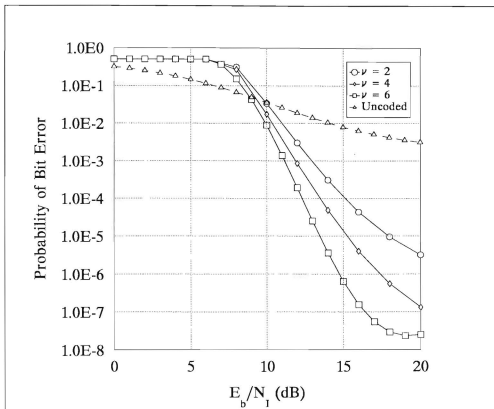


Fig. 13 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\nu = 2, 4$  and  $6$  under Rician fading, a strong direct signal ( $\alpha^2/2\sigma^2 = 10$ ), full band jamming,  $E_b/N_0 = 13.35\text{dB}$ , and  $L = 1$ .

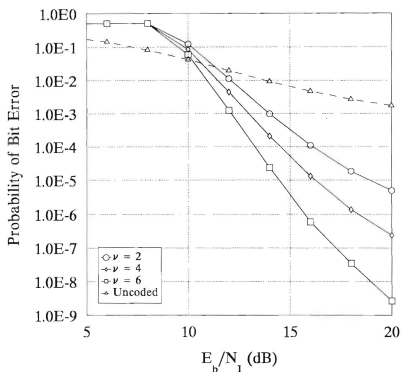


Fig. 14 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\nu = 2, 4$  and  $6$  under Rician fading, a strong direct signal ( $\alpha^2/2\sigma^2 = 10$ ), full band jamming,  $E_b/N_0 = 13.35$ dB, and  $L = 2$ .

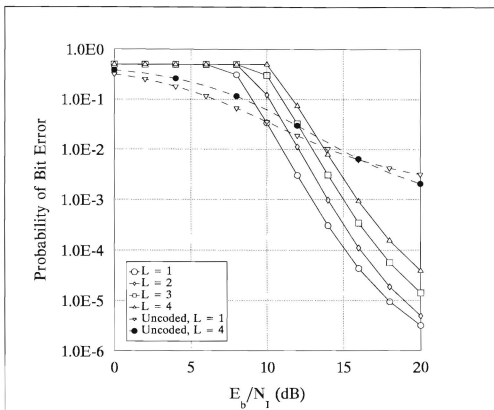


Fig. 15 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for number of hops/bit of  $L = 1, 2, 3$  and  $4$  with Rician fading, a strong direct signal ( $\alpha^2/2\sigma^2 = 10$ ), full band jamming,  $E_b/N_0 = 13.35\text{dB}$ , and  $v = 2$ .

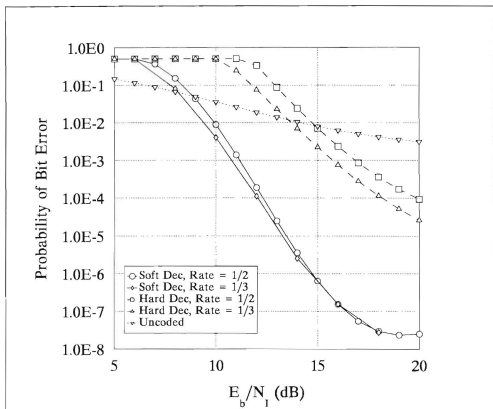


Fig. 16 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate,  $r = 1/2$  and  $1/3$  with Rician fading, a strong direct signal ( $\alpha^2/2\sigma^2 = 10$ ), full band jamming,  $E_b/N_0 = 13.35\text{dB}$ ,  $L = 1$ , and  $v = 6$ .

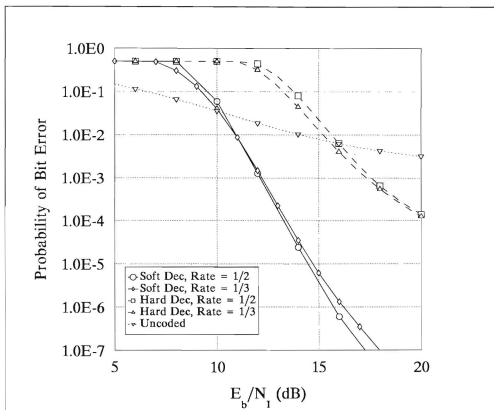


Fig. 17 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate,  $r = 1/2$  and  $1/3$  with Rician fading, a strong direct signal ( $\alpha^2/2\sigma^2 = 10$ ), full band jamming,  $E_b/N_0 = 13.35\text{dB}$ ,  $L = 2$ , and  $v = 6$ .

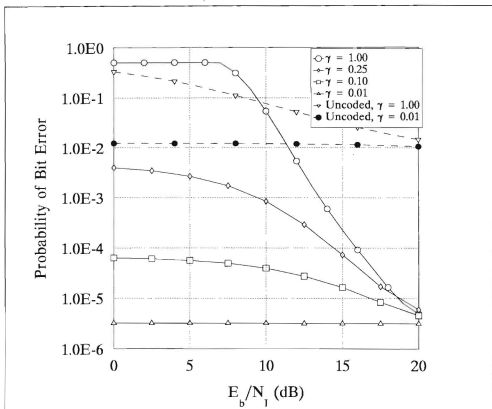


Fig. 18 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rician fading, a weak direct signal ( $\alpha^2/2\sigma^2 = 1$ ),  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 1$ , and  $v = 2$ .



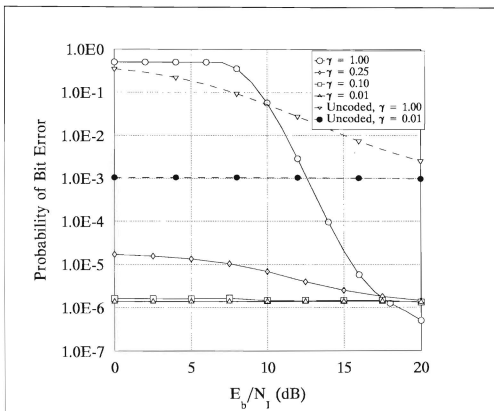


Fig. 19 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rician fading, a weak direct signal ( $\alpha^2/2\sigma^2 = 1$ ),  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 2$ , and  $\nu = 2$ .

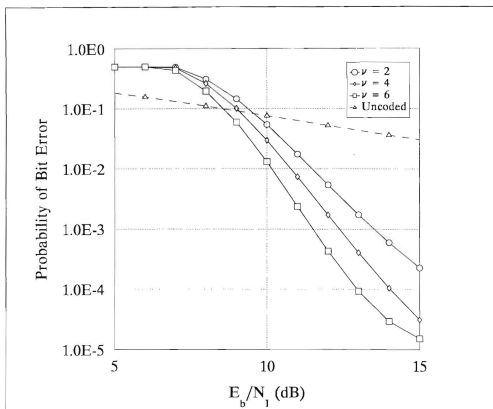


Fig. 20 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\nu = 2, 4$  and  $6$  with Rician fading, a weak direct signal ( $\alpha^2/2\sigma^2 = 1$ ), full band jamming,  $E_b/N_0 = 20.0\text{dB}$ , and  $L = 1$ .

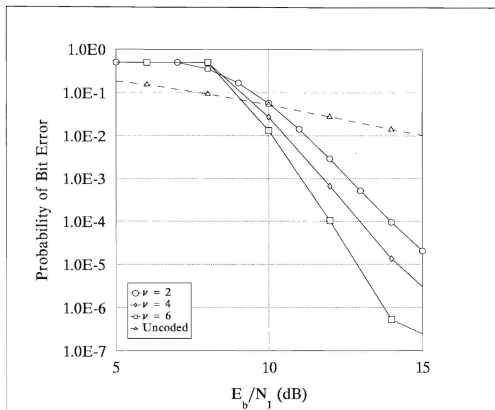


Fig. 21 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $v = 2, 4$  and  $6$  with Rician fading, a weak direct signal ( $\alpha^2/2\sigma^2 = 1$ ), full band jamming,  $E_b/N_0 = 20.0\text{dB}$ , and  $L = 2$ .

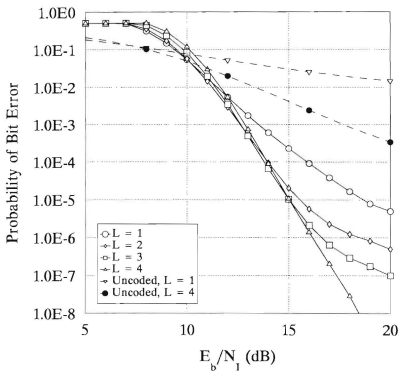


Fig. 22 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for number of hops/bit of  $L = 1, 2, 3$  and  $4$  with Rician fading, a weak direct signal ( $\alpha^2/2\sigma^2 = 1$ ), full band jamming,  $E_b/N_0 = 20.0\text{dB}$ , and  $\nu = 2$ .

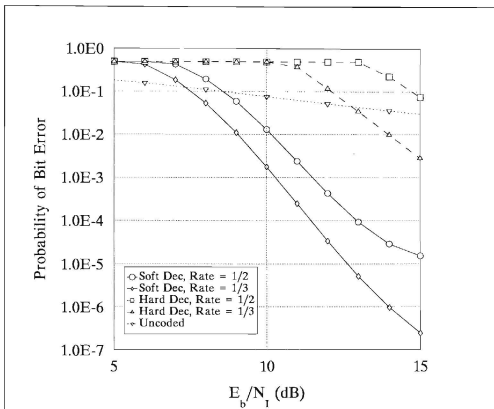


Fig. 23 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate,  $r = 1/2$  and  $1/3$  with Rician fading, a weak direct signal ( $\alpha^2/2\sigma^2 = 1$ ), full band jamming,  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 1$ , and  $v = 6$ .

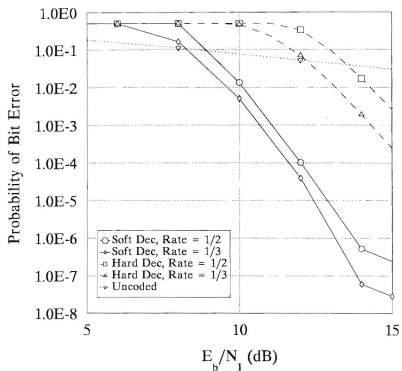


Fig. 24 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding and Hard-Decision Viterbi Decoding for code rate,  $r = 1/2$  and  $1/3$  with Rician fading, a weak direct signal ( $\alpha^2/2\sigma^2 = 1$ ), full band jamming,  $E_b/N_0 = 20.0\text{dB}$ ,  $L = 2$ , and  $v = 6$ .

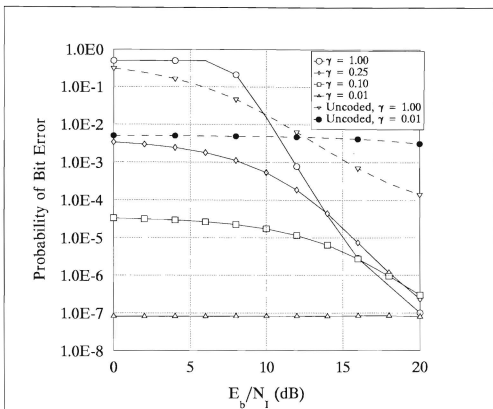


Fig. 25 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rician fading, a very strong direct signal ( $\alpha^2/2\sigma^2 = 100$ ),  $E_b/N_0 = 13.35\text{dB}$ ,  $L = 1$ , and  $v = 2$ .

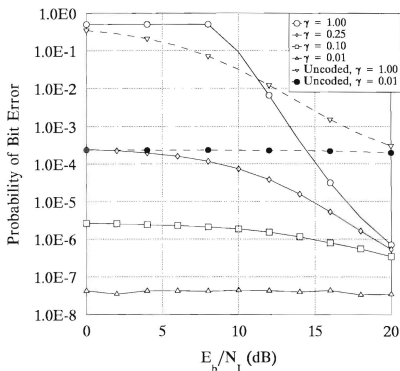


Fig. 26 : Performance of Noise-Normalization Receiver with Soft-Decision Viterbi Decoding for  $\gamma = 1.0, 0.25, 0.1$  and  $0.01$  with Rician fading, a very strong direct signal ( $\alpha^2/2\sigma^2 = 100$ ),  $E_b/N_0 = 13.35\text{dB}$ ,  $L = 2$ , and  $\nu = 2$ .



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